The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #2 ***Version 2.0***

Date: November 13, 2025 Course: EE 313 Evans

Name: ____
Last, First

- **Exam duration**. The exam is scheduled to last 75 minutes.
- **Materials allowed**. You may use books, notes, your laptop/tablet, and a calculator.
- **Disable all networks**. Please disable all network connections on all computer systems. You may not access the Internet or other networks during the exam.
- **No AI tools allowed**. As mentioned on the course syllabus, you may not use GPT or other AI tools during the exam.
- **Electronics.** Power down phones. No headphones. Mute your computer systems.
- **Fully justify your answers**. When justifying your answers, reference your source and page number as well as quote the content in the source for your justification. You could reference homework solutions, test solutions, etc.
- **Matlab**. No question on the test requires you to write or interpret Matlab code. If you base an answer on Matlab code, then please provide the code as part of the justification.
- **Put all work on the test**. All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Academic integrity.** By submitting this exam, you affirm that you have not received help directly or indirectly on this test from another human except the proctor for the test, and that you did not provide help, directly or indirectly, to another student taking this exam.

Problem	*Point Value*	*Your score*	*Topic*
1	27		System Properties
2	25		FIR Filter Analysis
3	24		System Identification
4	24		Upsampling, Downsampling & Filtering
Total	100		

Problem 2.1. *System Properties*. 27 points.

Each discrete-time system has input $x[n]$ and output $y[n]$, and $x[n]$ and $y[n]$ might be complex-valued.

Determine if each system is linear or nonlinear, time-invariant or time-varying, and causal or not causal.

You must either prove that the system property holds in the case of linearity, time-invariance, or causality, or provide a counter-example that the property does not hold. Providing an answer without any justification will earn 0 points.

Part	***System Name***	***System Formula***	***Linear?***	***Time-Invariant?***	***Causality?***
(a)	Averaging **Finite** Impulse Response Filter	$y[n] = x[n] + x[n+1]$ for $-\infty < n < \infty$			
(b)	Averaging **Infinite** Impulse Response Filter	$y[n] = 0.9\, y[n-1] + 0.1\, x[n]$ for $n \geq 0$			
(c)	Phase **Modulation**	$y[n] = \cos(\widehat{\omega}_0\, n + x[n])$ where $\widehat{\omega}_0$ is a constant for $-\infty < n < \infty$			

(a) Averaging Finite Impulse Response Filter: $y[n] = x[n] + x[n+1]$ for $-\infty < n < \infty$. *9 points*.

(b) Averaging Infinite Impulse Response Filter: $y[n] = 0.9\, y[n-1] + 0.1\, x[n]$ for $n \geq 0$. Analyze the impact of the initial condition(s) on the system properties. *9 points*.

(c) Phase Modulation: $y[n] = \cos(\widehat{\omega}_0\, n + x[n])$ where $\widehat{\omega}_0$ is a constant and $-\infty < n < \infty$ *9 points*.

Problem 2.2 *FIR Filter Analysis*. 25 points.

Consider the following causal finite impulse response (FIR) linear time-invariant (LTI) filter with input $x[n]$ and output $y[n]$ described by

$$y[n] = x[n] - a\, x[n-1]$$

for $n \geq 0$. Here a is a real-valued constant where $a \neq 0$.

(a) Give a formula for the impulse response $h[n]$. Plot $h[n]$. *3 points*.

(b) What are the initial condition(s)? What are their value(s)? *3 points*.

(c) Compute the transfer function $H(z)$ in the z-domain and give the region of convergence. *3 points.*

(d) Give a formula for the discrete-time frequency response of the FIR filter. *4 points*.

(e) Does the FIR filter have linear phase? If yes, then give the conditions on the coefficient a for the filter to have linear phase. If no, then show that the coefficients cannot meet the conditions for linear phase. *6 points*

(f) What are all of the possible frequency selectivities that the FIR filter could provide: lowpass, highpass, bandpass, bandstop, or allpass? *6 points*

Problem 2.3 *System Identification.* 24 points.

You're trying to identify unknown discrete-time systems.

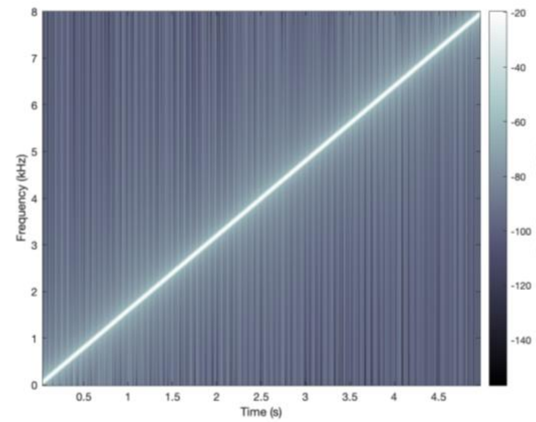
You input a discrete-time chirp signal $x[n]$ and look at the output to figure out what the system is.

The discrete-time chirp is formed by sampling a chirp signal that sweeps 0 to 8000 Hz over 0 to 5s

$$x(t) = \cos(2\pi f_1 t + 2\pi \mu t^2)$$

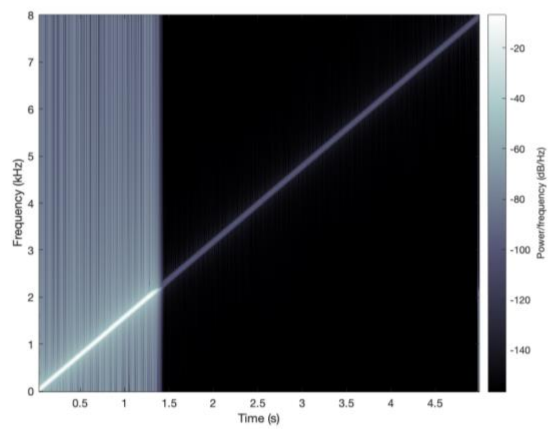
where $f_1 = 0$ Hz, $f_2 = 8000$ Hz, and $\mu = \frac{f_2 - f_1}{2\, t_{\max}} = \frac{8000 \text{ Hz}}{10\ s} = 800 \text{ Hz}^2$. Sampling rate f_s is 16000 Hz.

Spectrogram for chirp signal $x[n]$

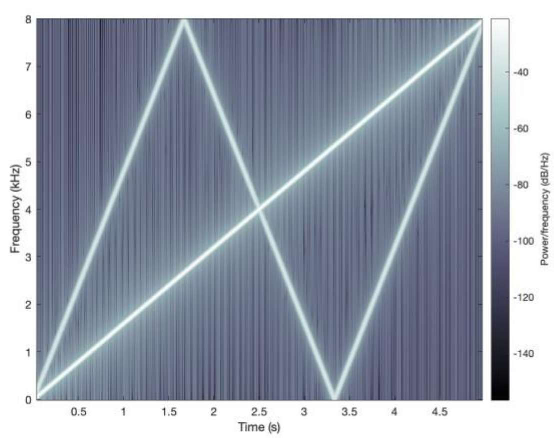


In each part below, identify the unknown system as one of the following **with justification**:

1. filter – give the frequency selectivity (lowpass, highpass, bandpass, bandstop) as well as the passband and stopband frequencies
2. pointwise nonlinearity – give the integer exponent k to produce output $y[n] = x^k[n]$
3. amplitude modulation – give the amplitude modulation frequency f_0 to produce output $y[n] = \cos(\omega_0\, n)\, x[n]$ where $\omega_0 = 2\pi\, f_0 \,/\, f_s$.



(a) When the chirp signal $x[n]$ is input, a system gives the output signal $y[n]$ whose spectrogram is plotted on the left. *12 points*.



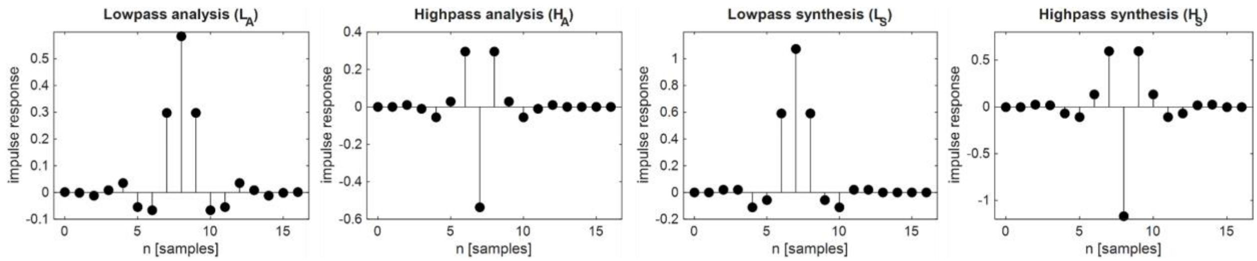
(b) When the chirp signal $x[n]$ is input, another system gives the output signal $y[n]$ whose spectrogram is plotted on the left. *12 points*

Hint: On the right of each spectrogram plot is an intensity map to decibels (dB). All values are negative.

Problem 2.4. *Upsampling, Downsampling, and Filtering.* 24 points.

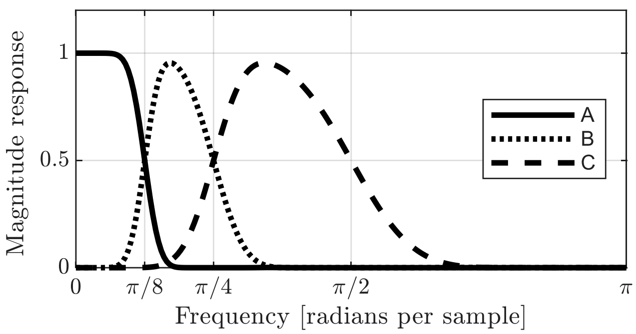
This problem is related to mini-project #2. Please justify your answers.

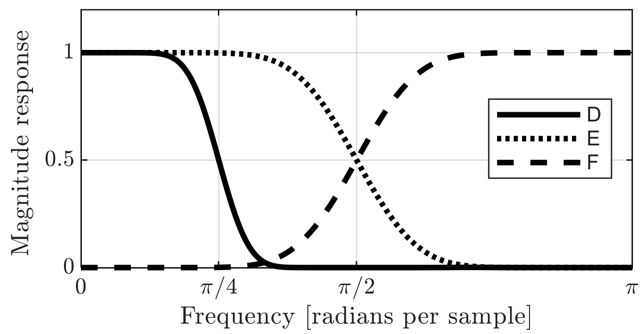
The impulse responses for four LTI filters L_A, H_A, L_S, and H_S are shown below. These filters are cascaded with downsampling ($\downarrow_2$) and upsampling ($\uparrow_2$) operations as shown in the block diagrams below.



i. Match each of the six system block diagrams with the magnitude responses (A-F) shown below. The magnitude response plot represents the change in magnitude when a complex sinusoid is input to the system, disregarding other frequencies that are created by upsampling.

ii. For each system, state whether or not the passband width is an octave. Assume that the passband is the set of frequencies where the magnitude response is greater than or equal to 0.5.





System Block Diagram	Match (A-F)	Is an octave?
$x[n] \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \hat{x}[n]$		
$x[n] \rightarrow \boxed{H_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{H_S} \rightarrow \hat{x}[n]$		
$x[n] \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \hat{x}[n]$		
$x[n] \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{H_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{H_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \hat{x}[n]$		
$x[n] \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \hat{x}[n]$		
$x[n] \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{L_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{H_A} \rightarrow \boxed{\downarrow_2} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{H_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \boxed{\uparrow_2} \rightarrow \boxed{L_S} \rightarrow \hat{x}[n]$		